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BEHAVIOR OF WATER-MODERATED REACTORS DURING RAPID TRANSIENTS

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BEHAVIOR OF WATER-MODERATED
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Merson Booth

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B.S., U. S. Naval Academy
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SUBMITTED IN PARTIAL FULFILLMENT OF THE

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BEHAVIOR OF WATER-MODERATED REACTORS DURING RAPID TRANSIENTS

by

Lieutenant Merson Booth, U.S.N.

Submitted to the Department of Chemical Engineering on 22 August 1955 in partial fulfillment of the requirements for the degree of Master of Science in Nuclear Engineering.

ABSTRACT

The recent increased number of proposals by institutions and industrial groups to build power or research reactors in populated areas has placed increased emphasis on reactor designs which are inherently safe against catastrophic reactor runaway accidents. Since water-moderated reactors can be designed with a large negative steam coefficient of reactivity, they possess inherent power-limiting characteristics which reduce the hazard of accidental nuclear runaway. Recently a series of intentional nuclear runaway experiments with a water-moderated reactor were conducted under the code name Borax to determine the reactor shutdown mechanism for a nuclear runaway excursion in a water-moderated reactor.

The purpose of the investigation described by this thesis has been to understand and analyze a nuclear runaway excursion in a water-moderated reactor. Toward this aim the results of the Borax and other boiling experiments were studied in detail. From this study a physical model of the reactor transient excursion was developed and certain phases of this model were analyzed in detail. The principal results of this analysis have been a prediction of the temperature profile in the water channels of the reactor prior to initiation of boiling, a criterion for the initiation of boiling and a good correlation of experimental maximum fuel plate temperatures for the Borax reactor.

More specifically, the temperature profile in a water channel prior to initiation of boiling is given by

$$T\ (x) - Ti = (T_p - Ti)\ e^{-x\sqrt{m/a}}$$

where T (x) is the water temperature in $^{\circ}$F at a distance x ft into the water, Ti is the initial water temperature, Tp is the fuel plate temperature, m is the reciprocal exponential reactor period in sec^{-1} and a is the heat diffusivity of water in ft^2/sec.

The general criterion developed for predicting when boiling will start is that the maximum amount of superheat energy contained in a hemispherical volume of water at the heated surface is sufficient to form a steam bubble of a particular size. When applied to water adjacent to a fuel plate whose temperature is increasing exponentially with a reciprocal period of m sec^{-1}, this general criterion becomes

$$0.1147 \sqrt{m} = \frac{(Tb - Ts)^{4/3}}{(Tb - Ti)}$$

where Tb is the plate temperature in $^{\circ}F$ at the time of initiation of boiling and Ts is the saturation temperature of water.

The correlation of experimental maximum fuel plate temperatures (T_{Pm}, $^{\circ}F$) for the Borax reactor is given by

$$0.715 \sqrt{m} = 3 \ell n (T_{Pm} - T_s) - 4/3 \ell n (Tb - Ts) - 5.82$$

where Tb, the fuel plate temperature at which boiling starts, is given by the preceding equation.

Graphs illustrating temperatures predicted by these equations and showing the agreement of observed and predicted temperatures are presented.

The secondary results of this thesis have been a qualitative description of the transient boiling phenomena and the reactor shutdown mechanism.

Thesis Supervisor: Manson Benedict

Title: Professor of Nuclear Engineering

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A. Introduction

A large fraction of both the initial and operating costs of present day power or research reactors stems from the hazards, both proven and hypothetical, which may beset the reactor. Principal among the reactor hazards in regard to increased protection costs is the hazard of accidental nuclear runaway. In water-moderated reactors the boiling process provides an inherent power limiting mechanism to reduce this hazard.

Utilization of the boiling process in water-moderated reactors to limit nuclear runaway depends upon the fact that such reactors can be designed to have a negative steam coefficient of reactivity. The formation of steam and displacement of water from the reactor causes a net decrease in reactivity such that the reactor may become subcritical in which case the power, after reaching a maximum, will decrease.

In the summer of 1953 a water-moderated reactor using MTR type fuel elements was constructed at the National Reactor Testing Station in Idaho to determine experimentally the self-limiting power characteristics of this type of reactor. During the summers of 1953 and 1954 a series of intentional nuclear runaway experiments were conducted under the code name Borax. A brief description of the Borax experiments is given in the following section.

The purpose of this thesis is to understand and analyze a nuclear runaway excursion in a water moderated reactor. Toward this aim the Borax and other boiling experiments were studied in detail. From this study a physical model of the reactor transient was developed and where feasible various phases of this model were analyzed in detail. The principal results of this analysis have been a prediction of temperatures in the reactor prior to initiation of boiling, a criterion for the initiation of boiling and an excellent correlation of experimental maximum fuel plate temperatures for the Borax reactor. Secondary results of this study have been a qualitative understanding of the transient boiling phenomena and the reactor shutdown mechanism.

B. Borax Experiments

Detailed descriptions of these experiments are given in References 1 and 2. The following brief description has been condensed from these reports.

1. Reactor Installation

The reactor installation is shown in a cutaway view in Figure 1. The reactor tank was contained in a larger shield tank of ten foot diameter which was sunk part way into the ground and had earth piled around it for additional shielding. Adjacent to the shield tank was a pit with concrete walls in which was installed equipment for filling and emptying the reactor and shield tanks, and for preheating the water in the reactor tank.

The reactor tank, which was four feet in diameter and about thirteen feet high, contained the reactor core, which consisted of a number of MTR type fuel elements held at the bottom by a supporting grid and at the top by a removable cover grid. The core grid could accommodate thirty-six fuel elements, but a maximum of thirty elements were used in the Borax program. In operation the reactor tank was filled with water to a height of three to four and one-half feet above the top of the core; this water constituted the reflector, moderator, and coolant.

2. MTR Fuel Elements

Figure 2 is a drawing of a standard MTR fuel element. Each element contained 18 fuel plates with a combined U^{235}

content (93% enrichment) of about 140 grams. The U^{235} in each plate is in the form of a strip of uranium-aluminum alloy, 23.6 inches long by 2.5 inches wide by 0.021 inch thick. The alloy plate was covered with a cladding of pure aluminum, which increased the total dimensions of the fuel plates to 24.6 inches by 2.845 inches by 0.060 inch. The water channel between the plates is 0.117 inch across.

3. Control Rods

The reactor contained five control rods; a central rod which was alternatively a flat plate or a cross-shaped member, as the requirements of the experiment dictated, and four wide flat plates (shim rods) which operated in the channels separating the four quadrants of the reactor core. All rods were made of nickel-clad cadmium in aluminum casings. The control rods were attached by extension rods to drive mechanisms located above the top of the reactor tank. The central rod was attached to its mechanism by an electromagnet, which when released allowed the rod to be spring-ejected downward out of the core for the experiments on reactor runaway.

4. Experimental Procedure

The experimental procedure was as follows: The fuel element loading of the reactor was adjusted to give approximately the amount of excess reactivity desired for the experiment. With shim rods fully inserted (reactor subcritical) the central control rod was inserted into the

core by an amount equal to the desired excess reactivity for the experiment. With the central rod held fixed at that position, the reactor was made critical at very low power (~ 1 watt) by withdrawal of the shim rods; the shim rods were then held at that critical position. The central rod could then be ejected from the core to produce a transient excursion.

5. Instrumentation

Three calibrated neutron counters were installed at various positions in the reactor to record the instantaneous neutron flux and power of the reactor. Thermocouples were attached to fuel plates in the vicinity of maximum flux in the reactor to record instantaneous fuel plate temperature. The control and recording instruments were located at some distance from the reactor proper for safety.

6. Results

The plot of a typical excursion is shown in Figure 3. The following observations are recorded after examination of a number of such experimental graphs:

a) The power increase with time is almost exponential up to approximately the time of maximum power. Deviation from the exponential could be attributed to temperature effects on reactivity.

b) Pressure remains at ambient pressure until approximately the time of maximum power and then

increases with time reaching a maximum at some time after maximum fuel plate temperature.

c) The temperature of an insulated fuel plate at any time is proportional to the time integral of the power to that time. Up to approximately the time of maximum power this curve is almost exponential.

d) The bare fuel plate temperature increases almost exponentially but dropping a little below the corresponding insulated plate temperature up to the time of maximum power, at which time the difference between the two temperature curves increases quite rapidly. The maximum fuel plate temperature occurs at some time after maximum power.

e) After the time of maximum power the power curve shows the effect of a steadily increasing negative period which indicates a steadily increasing steam volume in the reactor. At the time of maximum fuel plate temperature the power is decreasing quite rapidly.

Plots of experimental maximum fuel plate temperatures attained for excursions of various periods and initial subcooling are reproduced as Figures 4 and 5.

C. Previous Investigations Into the Boiling Phenomena

The results of several recent investigations of the boiling process will be used in this study.

Rosenthal (Ref. 3) has experimentally investigated the phenomena of initiation and subsequent behavior of boiling in water adjacent to a thin metal plate whose temperature is rising exponentially with time. The plate was electrically heated from an exponentially increasing power source. Plate temperatures were deduced from a continuous record of plate electrical resistance. Motion pictures were taken of the plate area at the rate of 6000 frames per second. Runs were conducted for various initial water temperatures and exponential periods.

The experimental results of this investigation as briefly presented in a preliminary report (Ref. 3) are as follows:

1. The rapidly rising plate temperature passes the saturation temperature of water and before boiling commences exceeds it by an amount termed the "temperature overshoot."
2. Suddenly there is an almost explosive formation of bubbles covering the surface.
3. This boiling surge expires and for a moment the surface is nearly free of bubbles.
4. Then boiling commences which is similar in appearance to local boiling with steady generation of heat.

5. With initial water temperatures above 190° F for the range of periods studied the temperature overshoot is less than 6° F within the accuracy ($\pm$ 6° F) of the temperature measurements.
6. Data given for one run showed that for an exponential period of 17 milliseconds and an initial temperature of 92° F the plate temperature at the time of initiation of boiling was 253° F and the time duration of the initial boiling surge was approximately 5 milliseconds.

Whitehead (Ref. 4) has found from photographs taken of boiling in water under a wide range of conditions that plots of number of bubbles observed of a given radius versus the radius show a rather sharp peak at a uniform radius of approximately 3 mils.

Rohsenow (Ref. 5) has shown that an excellent correlation of experimental boiling heat transfer data can be made with the equation

$$\frac{Q}{A} = \frac{c_l^3 \mu_l}{C_{sf}^3 \, h_{fg}^2 \sqrt{\frac{2 g_0 \sigma}{g(\rho_l - \rho_v)}} \left(\frac{c_l \mu_l}{k_l}\right)^{5.1}} \left(T_p - T_s\right)^3$$

where Q/A is the rate of heat transfer per unit area, T_p is the surface temperature, and T_s is the boiling temperature of water. C_{sf} is a constant for any surface material-fluid combination. The remaining quantities are properties of water or universal constants and are defined in Appendix A.

The most important result of this correlation for this thesis is that the heat flux to boiling water (Q/A) is proportional to the cube of the difference in temperature between metal surface (T_p) and boiling water (T_s).

D. Sequence of Physical Events in a Reactor Excursion

Consideration of the Borax experimental results and these recent investigations into the boiling process lead one to divide the reactor excursion into a number of phases representing the time sequence of physical events occurring within the reactor. The reader is asked to focus his attention on one fuel plate and adjacent water channel. Figure 6 gives a pictorial representation of the assumed temperature profiles for a fuel plate and adjacent water during each phase.

1. Conduction Phase

In the initial stages of a rapid reactor transient prior to the initiation of boiling the only mechanism for transferring heat produced in the fuel plates to the water is conduction. There is not time for natural convection to become effective in transferring heat. The power and fuel plate temperature are increasing approximately exponentially during this time, and there exists at any time a temperature gradient in the water which can be predicted by solution of the differential equation for transient heat conduction in metal and water.

2. Initiation of Boiling

At the time when the fuel plate temperature and the energy transferred to the water have reached critical values to be derived in this thesis, water at the plate surface will erupt in an almost explosive formation of bubbles.

3. Underdeveloped Boiling Phase

The subsequent agitation of the water by boiling incorporates subcooled water in the boiling region. During this phase the rate of heat transfer into this region from the plate is less than the rate of heat transfer out of the region by turbulent mixing with subcooled water. The average water temperature and the degree of boiling decrease until the surface is almost free of bubbles.

4. Fully Developed Boiling

As the temperature of the fuel plate continues to increase, underdeveloped boiling gives way to fully developed boiling. Fully developed boiling is defined as that condition for which the rate of heat transfer into the boiling region equals that out of this region such that there will be no net increase in vapor formation with time. At this time heat transfer out of the boiling region is by conduction into the subcooled water.

5. Overdeveloped Boiling Phase

If the fuel plate temperature continues to increase, the rate of heat transfer into the boiling region will become greater than that conducted out and the net rate of vapor formation will increase with time. This is known as overdeveloped boiling. The process is more or less self-limiting since the increased bubble agitation and turbulent mixing of subcooled water can quickly diminish the rate of increase of vapor formation.

6. Outward Progress of Boiling in the Reactor

The above sequence of events for a single fuel plate and water channel starts first at the point of maximum flux, temperature and power density within the reactor. The identical sequence of events, displaced later in time, occurs in other channels which are displaced in distance from the channel of maximum flux. The time displacement or lag in appearance of this sequence of events in any water channel depends on the time displacement of flux and temperature at this point relative to the point of maximum flux.

Commencing with the initiation of boiling at the point of maximum flux, steam is being produced within the reactor in increasingly larger quantities. Due to the negative steam coefficient of reactivity the effective multiplication factor of the reactor decreases with time.

E. Scope of Thesis

The conduction phase, the condition for initiation of boiling, and the condition of fully developed boiling lend themselves to analytical investigation. This thesis presents a detailed analysis of these phases of a reactor excursion in regard to plate temperature, rate of heat transfer, and growth of the temperature gradient in the water. This analysis leads to a good correlation of maximum fuel plate temperatures observed in the Borax experiments.

The remaining phases of a reactor excursion are more difficult to treat analytically. Fundamental knowledge of the processes occurring during the phases of underdeveloped and overdeveloped boiling which is not now available is required. The thesis gives only a qualitative description of the sequence of events occurring during these phases. Quantitative analysis of the spread of boiling through the reactor requires the aid of analog computing equipment and is beyond the scope of this thesis.

F. Results of Analysis

1. Conduction Phase

The following physical model of the conduction phase is proposed:

a) The reactor is originally just critical at very low power.

b) The water is stagnant within the water channels.

c) The water and fuel plates are initially at some uniform temperature.

d) There are no natural convention currents set up in the water channels during the short time to reach maximum power.

e) There is no boiling or vapor formation during this phase.

f) Heat transfer from the fuel plates to the water during this phase is by transient conduction.

g) The diffusivity of heat through the fuel plates is much larger than that through the water. Therefore there is essentially no temperature gradient in the fuel plates; the gradient all being in the water.

h) The amount of energy transferred to the water during this phase is small compared with the energy contained in the fuel plates.

Conditions (a) through (e) are those present in the Borax reactor. Condition (f) is a consequence of conditions (d) and (e). Condition (g) is true for aluminum plates and water. Condition (h) is open to serious objection. Although the temperature of an insulated fuel plate increases approximately exponentially with time the temperature of a real fuel plate drops below this insulated plate temperature by an amount proportional to the energy transferred to the water. For the short reactor periods being dealt with here this temperature difference between the insulated and real fuel plates is small. But, a prediction of fuel plate temperature as a function of time will be in error if it assumed that the temperature increases exponentially with the reactor period. However, since we relate the solution for the temperature gradient in the water to the fuel plate temperature instead of to a function of time the objection to this condition is partially nullified.

The reactor kinetics (prompt neutrons only and ignoring secondary temperature effects on reactivity) for this phase are described by

(Table of symbols is given in Appendix A.)

(1) $$\frac{dP}{dt} = Pm$$

where

(2) $$m = \frac{\delta k - \beta}{\ell}$$

Integrating equation (1) between Pi at t = 0 and P at t = t gives

(3) $$P = Pi\, e^{mt}$$

and for the relation between the rate of temperature rise in the fuel plates and the power, we have

$$(4) \qquad S \frac{dT_p}{dt} = P$$

Substituting equation (3) into equation (4) and integrating from $T_p = Ti$ at $t = 0$ to $T_p = T_p$ at $t = t$ gives

$$T_p - Ti = \frac{Pi}{Sm} (e^{mt} - 1)$$

Disregarding the 1 as insignificant during the greater part of this phase in respect to e^{mt}, we get

$$(5) \qquad T_p - Ti = \frac{Pi}{Sm} e^{mt}$$

Since during this conduction phase heat has penetrated only the order of a few mils into the water we can consider the water to be a semi-infinite medium with a surface temperature given by equation (5). Solving the transient problem of diffusion of heat into a semi-infinite medium with an exponentially increasing surface temperature (Appendix B) we find the asymptotic solution after the initial transients have died out, for the temperature of the water a distance X from the surface at time t to be

$$(6) \qquad T \ (x,t) - Ti = \frac{Pi}{Sm} e^{mt} e^{-x\sqrt{m/a}}$$

or at the time the plate temperature is T_p

$$(7) \qquad T \ (x,T_p) - Ti = (T_p - Ti) e^{-x\sqrt{m/a}}$$

Note that equations (5) and (6) are somewhat in error in predicting the time variation of temperature due to condition (h) of the basic assumptions for this phase, but equation (7) is much less in error than (6) from this cause. Figure 7 pictorially shows the changing temperature gradient existing during this phase.

2. Initiation of Boiling

The following physical model for a criterion for the initiation of boiling is proposed:

a) For a given plate temperature T_p, initial temperature Ti, and inverse period m, the temperature gradient into the water is given by equation (7).

b) A prospective bubble will grow from a point on the plate surface.

c) A prospective bubble, to grow, must draw upon energy above saturation temperature contained in the water surrounding it.

d) The prospective bubble or point on the plate surface has the ability to "look" out in the surrounding water in a more or less hemispherical fashion, and to itself integrate the energy above saturation temperature contained in hemispheres of varying radii.

e) Because of the temperature gradient in the water the integrated energy will at first increase with

Note that equations (3) and (4) are identical in [illegible] in predicting time-lag variations of temperature for the condition (b) of the above assumptions for thin plates, but equation (7) is much less in error than (6) from this [illegible]. **Figure 7** [illegible] shows the changing temperature [illegible] existing during thin plates.

8. Mechanism of Cracking

The following physical model for a [illegible] thermal mechanism of cracking is proposed:

a) For a given plane temperature T_1, initial temperature T_0, and [illegible] the temperature gradient into the solid is given by equation (2).

b) A temperature pattern will arise from a point on the plate surface.

c) A temperature pattern, in turn, [illegible] thermal stress [illegible] temperature [illegible] (M).

d) The temperature [illegible] at a point on the plate surface [illegible] in "time" and in the surrounding region in a more or less [illegible] fashion, and [illegible] temperature [illegible] of [illegible].

e) Because of the temperature gradient in the [illegible] stress will at first increase with increasing radii as larger volumes of superheated water are enveloped, then decrease as subcooled water is enveloped. Thus, there is a maximum energy above saturation temperature contained in a hemispherical volume of water under these conditions.

f) The prospective bubble will have available to it the maximum energy above saturation temperature contained in the hemispherical volume of water.

g) The prospective bubble will form only if there is energy available for it to grow to a given radius, of the order of 3 mils.

Condition (f) should be quite good for the case of a steep temperature gradient into the water since the radius of the hemispherical volume of water containing the maximum energy is small and this energy is closer and more readily available to the bubble. For the case of small values of subcooling the temperature gradient into the water is flatter and the radius of the hemispherical volume of water containing the maximum energy is larger and this energy is less readily available for formation of the bubble. Thus the criterion to be developed from this model will be in error in the cases of small values of subcooling.

Condition (e) appears at first to be rather arbitrary. Whitehead (Ref. 4) has found from photographs taken of boiling in water under a wide range of conditions that plots of

number of bubbles observed of a given radius versus the radius show a rather sharp peak at a uniform value of approximately 3 mils.

Using a linear approximation to the exponential water temperature gradient (mathematical details are in Appendix C) results in a criterion for the plate temperature at the time of initiation of boiling, Tb, given by

$$(8) \qquad B\sqrt{m} = \frac{(Tb - Ts)^{4/3}}{(Tb - Ti)}$$

Here Ts is the boiling temperature of water, Ti is the initial temperature of the water, and m is the reciprocal of the period of the exponential rise of plate temperature. B depends only on the pressure and the assumed bubble radius (see equation (56) Appendix C). The temperature gradient at the initiation of boiling is shown pictorially in Figure 7.

Substitution of water properties at 212° F and one atmosphere and use of an assumed radius of 3 mils results in a calculated value of B

$$B = 0.1271^{\circ}\ F^{1/3}\ Sec^{\frac{1}{2}}$$

B can be evaluated experimentally from Rosenthal's one reported experiment (Ref. 3). This value at atmospheric pressure is

$$B = 0.1147^{\circ}\ F^{1/3}\ Sec^{\frac{1}{2}}$$

This close agreement provides strong support for the assumed condition for bubble formation.

Using this latter value of B, (Tb - Ts) is plotted as a function of (Ts - Ti) and m in Figures 8 and 9. Remembering that for small values of (Ts - Ti) this criterion is not very accurate and from the results of analysis in a later section in comparison with Borax temperature data, it will be assumed that (Tb - Ts) approaches an asymptote of 6° F as (Ts - Ti) approaches zero. The curves in this region are skewed to satisfy this requirement. This is not to say that in steady state boiling of essentially saturated water the plate temperature will be 6° F above saturation temperature. What is meant is that in cases of rapid transients as being dealt with here it appears from the data that this asymptotic value exists. Rosenthal (Ref. 3) found that for the exponential periods he investigated with Ti above 190° F the value of (Tb - Ts) was not over 6° F within the accuracy (± 6° F) of his temperature measurements.

3. Underdeveloped Boiling Phase

At the moment boiling begins there is present in the water a large amount of superheat. Initially the driving force for vapor formation is this superheat energy and the initial formation of vapor is almost explosive (Ref. 3) in character. However, the subsequent bubble agitation and turbulent mixing incorporates subcooled water into the region of boiling which reduces the average temperature, pressure and degree of boiling in that region.

Certainly, during this phase heat is being transferred from the plate to the boiling region. But as vapor formation increases during the first stages of this phase the pressure and therefore the saturation temperature increase, thus reducing the effective boiling heat transfer to this region from the plate. This phase is characterized by the fact that the rate of heat transfer into the boiling region is less than that out (primarily turbulent inflow of subcooled water). The net result is that the degree of boiling decreases to a point of almost no bubbles present in the later stages of this phase (Ref. 3).

In the latter stages of this phase the degree of boiling and consequently the pressure and saturation temperature in this channel decrease. At the same time the plate temperature is increasing so that the rate of boiling heat transfer from the plate to this region is increasing while the rate of convective heat transfer out of this region is decreasing.

Physically, the underdeveloped boiling phase is a transition phase from the condition of fully developed conduction to the condition of fully developed boiling.

4. Fully Developed Boiling

Fully developed boiling is defined as that condition for which the rate of heat transfer into the boiling region equals that out of this region so that there will be no net increase in vapor formation with time. The temperature profile for this condition is pictorially shown as curve 4 of Figure 7. This condition may, but does not necessarily have to, occur in any given reactor. If the reactor is very rapidly shut down during the outward progress of initial boiling (say in the case of a flat flux reactor), then the power and thus the rate of plate temperature rise may be very small and the under-developed boiling phase may not develop to the fully developed boiling condition. In the Borax reactor it appears as though the fully developed boiling condition is attained at least at the point of maximum flux. Certainly in all reactors this condition is not attained in all channels which underwent initial boiling.

With no net increase of steam and a very low level of the degree of boiling for the fully developed boiling condition, the amount of turbulent mixing is very small. Thus the heat transfer out of the boiling region will be principally by conduction into the subcooled water. Since the degree of boiling at this time is small, the pressure in the water channel has returned to essentially

ambient pressure and the temperature in the boiling region is essentially saturation temperature at ambient pressure. If we knew what the temperature gradient was at this time in the water at the outer edge of the boiling region we could evaluate the rate of heat transfer out of the boiling region.

I will postulate that the temperature profile into the subcooled water at the time of fully developed boiling bears some relation to the temperature profile which existed in the water at the time at which boiling started. More specifically, it is postulated that the slope of the temperature profile in the water at the outer edge of the boiling region at the time of fully developed boiling is equal to the slope of some point on the temperature profile which existed at the time boiling started.

At the instant of bubble formation the temperature gradient in the water was given by

$$(9) \quad (T_{\ell (x)} - T_i) = (T_b - T_i)\, e^{-x\sqrt{\frac{m}{a}}}$$

and the rate of heat transfer per unit area through the water by conduction at any point x is given by

$$(10) \quad \left(\frac{Q}{A}\right)_x = -K_\ell \left(\frac{d\, T_{\ell(x)}}{d\, x}\right)_x = K_\ell \sqrt{\frac{m}{a}}\, (T_b - T_i)\, e^{-x\sqrt{\frac{m}{a}}}$$

According to the above postulate, the slope of the temperature gradient at the outer edge of the boiling region at the time of fully developed boiling is equal to the slope of the temperature gradient at the instant of bubble formation given by equation (9) at some position along this gradient, $x = x'$. Note that here the quantity x' is defined as the distance from the plate into the water at which the slope of the temperature gradient which existed at the instant of bubble formation equals the slope of the temperature gradient at the outer edge of the boiling region at the time of fully developed boiling. The physical definition of x' is clear, although the manner in which it might vary with period and subcooling is not defined at present. Figure 7 shows a schematic representation of the temperature gradient existing at this time and the distance x' which is experimentally shown latter to be a virtual distance.

Thus the rate of heat transfer per unit area leaving the boiling region at its outer edge at the time of fully developed boiling is given by

$$(11) \quad \left(\frac{Q}{A}\right)_{OUT} = K_l \sqrt{\frac{m}{a}} \left(T_b - T_i\right) e^{-x'\sqrt{\frac{m}{a}}}$$

The rate of heat transfer per unit area from the fuel plate to the water in boiling is given by Rohsenow (Ref. 5) to be

(12) $$\left(\frac{Q}{A}\right)_{BOILING} = \alpha \left(T_P - T_S\right)^3$$

where

(13) $$\alpha = \frac{C_\ell^3 \mu_\ell}{C_{sf}^3 h_{fg}^2 \sqrt{\frac{2 g_0 \sigma}{g(\rho_\ell - \rho_v)}} \left(\frac{C_\ell \mu_\ell}{K_\ell}\right)^{5.1}}$$

Rohsenow gives values for C_{sf} for various combinations of surface material and fluid to range from 0.003 to 0.015. Evaluating the expression for α for water boiling at atmospheric pressure we get

$$\alpha = \frac{3.61 \times 10^{-6}}{C_{sf}^3} \; \frac{\text{BTU}}{\text{hr ft}^2 \; {}^\circ\text{F}^3}$$

or for the range of surfaces used by Rohsenow

$$\alpha = 1.07 \text{ to } 134 \; \frac{\text{BTU}}{\text{hr ft}^2 \; {}^\circ\text{F}^3}$$

This uncertainty in the value of α by a factor of 125 or more is disconcerting when attempting to carry through numerical calculations. However, the mere fact that it is a constant for a given pressure and surface-liquid combination is helpful since if it can be once evaluated this value may be used in further calculations involving this same surface-liquid combination.

Applying the definition of fully developed boiling, that the net rate of heat transfer to the boiling region

is zero, we get from equations (11) and (12)

$$(14) \qquad \alpha (T_P - T_S)^3 = K_\ell \sqrt{\frac{m}{a}} (T_b - T_i) \, e^{-x' \sqrt{\frac{m}{a}}}$$

This equation should predict the plate temperature at the instant of fully developed boiling.

The reader is now asked to focus his attention on the fuel plate temperature at the time of fully developed boiling. The rate of change of fuel plate temperature at any time is given by the following differential equation.

$$(15) \qquad S \frac{dT_P}{dt} = P - \left(\frac{Q}{A}\right)_{\substack{\text{PLATE TO} \\ \text{WATER}}}$$

Observations of the Borax data (Figure 3) show that at the time of maximum fuel plate temperature the power is decreasing quite rapidly. And from the discussion of the underdeveloped boiling phase it will be remembered that in the later stages of this phase the heat transfer rate from the plate to the water is increasing with time. Consideration of these facts in connection with equation (15) suggests that the time rate of change of plate temperature may pass through zero close to the time of attainment of fully developed boiling for the Borax reactor. At least the time rate of change of plate temperature will be very small at this time. I will therefore postulate that for the Borax reactor maximum fuel plate temperature occurs at the time of attainment of fully developed boiling.

With this postulate, shifting attention back to equation (14) we can write for the Borax reactor

$$(16) \quad \alpha \left(T_{Pm} - T_S \right)^3 = K_\ell \sqrt{\frac{m}{a}} \left(T_b - T_i \right) e^{-x'\sqrt{\frac{m}{a}}}$$

where T_{Pm} is the maximum plate temperature which is assumed to be reached simultaneously with fully developed boiling. Equation (16) together with equation (8) will be used to predict maximum fuel plate temperatures for the Borax reactor.

Rohsenow's experiments show that α should be independent of period (m) and initial degree of subcooling (Ts - Ti); I shall assume that x' is also independent of these variables.

Taking the following values for water properties:

$$(17) \quad a = 1.322 \times 10^{-3} \text{ ft sec}^{-\frac{1}{2}}$$

$$(18) \quad K_\ell = 0.394 \frac{\text{BTU}}{\text{hr ft } ^\circ\text{F}}$$

and the value determined for B from Rosenthal's data, comparison with the experimental Borax data for maximum fuel plate temperature shows an excellent fit of the data is obtained if one uses

$$(19) \quad x' = -0.945 \times 10^{-3} \text{ ft} = -11.35 \text{ mils}$$

and

$$(20) \quad \alpha = 4.15 \times 10^{-3} \frac{\text{BTU}}{\text{Sec ft}^2 \, ^\circ\text{F}^3} = 14.96 \frac{\text{BTU}}{\text{hr ft}^2 \, ^\circ\text{F}^3}$$

The result of substituting these values into equations (8) and (16) is

$$(21) \qquad 0.1147\sqrt{m} = \frac{(Tb - Ts)^{4/3}}{(Tb - Ti)}$$

$$(22) \qquad 0.715\sqrt{m} = 3\,\ell n\,(T_{Pm} - T_s) - \frac{4}{3}\,\ell n\,(Tb - Ts) - 5.82$$

where the temperatures are in $^{\circ}F$ and m is in sec^{-1}.

These equations constitute the correlation of maximum fuel plate temperatures in the Borax experiments. Using these equations, predictions of $(T_{Pm} - T_s)$ as a function of $\sqrt{m}$ and (Ts - Ti) for the Borax reactor were calculated and plotted in Figure 10.

For the case in which the water was initially saturated the best fit was obtained if Tb - Ts = 6 °F for all periods. This was the basis for the requirement that Tb - Ts approach an asymptote of 6 °F as Ts - Ti approached zero as explained in the section in which the criterion for boiling was presented.

A number of Borax experimental maximum plate temperature values from Figure 4 are replotted as circles in Figure 10 to show the agreement between experiment and prediction for the initially saturated case.

The Borax experimental maximum plate temperatures from Figure 5 for various degrees of subcooling and for 13 and 22 milisecond periods are replotted as circles in Figure 11.

The corresponding predicted maximum plate temperatures for these two periods are shown as solid lines in Figure 11. The close agreement shows the validity of the correlation of maximum plate temperatures by equations (21) and (22).

5. Overdeveloped Boiling Phase

After the establishment of fully developed boiling, a given water channel may go into the overdeveloped boiling phase in which the rate of heat transfer into the boiling region is greater than that out of this region. Thus the rate of vapor formation in this channel will increase with time. However, this process is more or less self limiting since the boiling agitation incorporates more subcooled water in the region thus reducing the rate of vapor formation as explained previously for the underdeveloped boiling phase.

It appears from the analysis in the preceding section that the Borax fuel plate temperatures did not increase significantly after the fully developed boiling condition. Therefore it appears that the Borax reactor didn't go very far into the overdeveloped boiling phase.

Under other conditions, however, overdeveloped boiling may take place to a significant extent before maximum plate temperatures are reached. Such could be the case in a heavy water reactor.

6. Outward Progress of Boiling in the Reactor

The reader is now asked to shift his attention from the sequence of physical events occurring within a single water channel to a consideration of these events occurring successively in various water channels in the reactor.

Boiling first commences within a water channel nearest the point of maximum flux in the reactor. The boiling region in that initial channel will rapidly progress axially in both directions from the initial point until some fraction of the fuel plate area bounding this channel is in boiling. At this time the boiling area will reach a maximum in this channel due to the combined effects of: (1) lower flux and the associated lower plate temperature at this distance from the point of maximum flux, (2) rapid vapor formation in the boiling region increases the pressure and the saturation temperature at all points in the channel, and (3) this rapid vapor formation and increased pressure forces the water out both ends of the channel, the rapid movement of which causes turbulent mixing of the water downstream thus partially destroying any temperature gradient near the wall which had been established in this water.

In the meantime, however, further water channels circumferentially surrounding the initial channel undergo initial boiling in a manner similar to that described for

the initial channel. (See Figure 6 for a pictorial representation of the outward progress of boiling in the reactor.) As further initial boiling takes place the power developed by the reactor varies in accordance with the following differential equation:

$$(23) \quad \frac{dP}{dt} = P\left(\frac{\delta K - \beta - \int_v \omega V_v \left(\frac{\partial K}{\partial V}\right) dA}{\ell}\right)$$

where P is the instantaneous power produced by the reactor, t is time, δK is the initial step increase in effective multiplication factor of the reactor, β is the fraction of fission neutrons which are delayed, ℓ is the prompt neutron lifetime, ω is the statistical weight at a point for reactor perturbations, V_v is the volume of steam per unit fuel plate area, $\frac{\partial K}{\partial V}$ is the reactor void coefficient and A is the fuel plate area.

The time sequence for initial boiling in various water channels within the reactor depends on the time required for the fuel plate temperature adjacent to these channels to reach T_b. If ϕ_o is the maximum flux, ϕ_r is the flux at point r, and t is the time lag between initial boiling in the channel at position r and initial boiling in the channel of maximum flux, the following equation approximately describes the interval of time which elapses between initial boiling at position of maximum flux and position r.

$$(24)\qquad T_b - T_i = \frac{\phi_r}{\phi_o}\left[(T_b - T_i) + \frac{1}{S}\int_0^t P_o(t)\,dt\right]$$

Consider for a moment the initial formation of vapor in any water channel. The criterion developed for the initiation of boiling contained the assumption that the superheat energy in the water available for vapor formation is a constant at the time of initiation of boiling. Thus the initial volume of vapor produced in the initial surge of boiling in any water channel from this constant available energy source is also a constant essentially independent of subcooling or period. In addition, the process of initiation of boiling in all water channels is identical essentially independent of subcooling or period.

Thus it will be assumed that the initial volume of steam formed in any water channel is a constant and for times shortly after the initiation of boiling in the central channel the total volume of steam in the reactor is this constant steam volume per channel multiplied by the number of channels undergoing boiling.

If the flux and water channels could be considered to be axially symmetric about the central channel and initial boiling with a constant vapor volume per channel is taking place inside a group of water channels describing a circle of radius r, viewing a transverse section of the reactor through the point of maximum flux, then the total vapor volume in the reactor is given by

$$(25) \qquad \int_V Vv \, dA = Cr^2$$

where C is a constant for any given fuel plate-water channel geometry and dimensions.

Equations (23), (24), and (25) can be solved, probably easier on a computing machine, revealing the course of the shutdown mechanism during the period shortly after the time of initiation of boiling in the first channel.

In deriving equation (25) it has been assumed that the volume of steam formed in any channel at the initiation of boiling in that channel does not change as boiling moves outward from this point, whereas in sections F3 and F5 it was assumed that the vapor volume decreases during the underdeveloped boiling phase and then increases during the overdeveloped boiling phase. It appears from consideration of the Borax excursions (Figure 3) that the outward progress of initial boiling in the reactor has accomplished the major contribution to the shutdown process before the attainment of fully developed boiling and a minimum of vapor volume in the central channel. The effect of this decrease in vapor volume will first be felt in the central channel. But the contribution of the vapor volume assumed to be in this central channel is small in comparison with the vapor volume contained in the

larger number of channels at radius r. Therefore the error made in not considering this decrease in vapor volume in the central channel shortly after initiation of boiling will be small.

Thus the suggested model for the shutdown process as given by equations (23), (24) and (25) approximately describes the reactor shutdown mechanism shortly after the initiation of boiling in the central channel. For the Borax reactor the outward progress of initial boiling in the reactor contributed much more to the reactor shutdown mechanism than did the secondary growth of vapor in the central and adjacent channels.

G. Discussion of Results

The principal results of this study have been the prediction of temperatures existing during the conduction phase, the criterion for initiation of boiling, and the correlation of experimental maximum fuel plate temperatures for the Borax reactor. Secondary results of this study have been a better, but still qualitative, understanding of the transient boiling phenomena and the reactor shutdown mechanism.

In the conduction phase it should be noted that although the time relation of temperatures is not known with accuracy the relation between temperatures in the water and fuel plate temperature is much more accurately known. In this regard, although the energy developed by the reactor, which is the time integral of the instantaneous power, is known at the time of first initiation of boiling (approximately the point of maximum power for the Borax reactor), the value of the instantaneous power at this time is not known with accuracy.

The criterion for initiation of boiling developed in this thesis has not been fully confirmed by experiment. However, two facts lend credulity to the criterion. First, numerical evaluation of the quantity B (equation 8) from water properties and the assumed bubble radius of 3 mils agrees remarkably well with an experimental evaluation of B from Rosenthal's one reported experiment. And second, the

criterion was used with success in connection with the correlation of experimental maximum fuel plate temperatures for the Borax reactor.

It is thought that the derived correlation may be used to quite accurately predict maximum fuel plate temperatures encountered in rapid transients of Borax type reactors. However, it is probably only a fortunate accident that for the Borax reactor the time of maximum fuel plate temperature happened to correspond very closely with the attainment of fully developed boiling in the central channel.

Another valuable result of this correlation has been an experimental determination of α, the heat transfer coefficient for boiling of water in contact with MTR fuel elements.

Still in the realm of speculation is the result from this correlation that the quantity x' is a constant. Since this quantity represents some unknown (at the present time) measure of the physical processes of bubble initiation and underdeveloped boiling phase, the fact that it is a constant leads one to believe that these processes are physically similar irrespective of period or degree of subcooling.

This belief is the basis for the suggested model of the shutdown mechanism shortly after the initiation of boiling given in section F5, dealing with outward progress

of initial boiling in the reactor.

Mention should be made of the apparent inconsistency between the assumed model of transient boiling and the Borax data regarding observed pressures. In the initial stages of underdeveloped boiling it was presumed that the pressure in a water channel increases with time. And in the later stages of this phase it was presumed that the pressure decreases with time. The attainment of the condition of fully developed boiling was marked by a return to essentially ambient pressure within the channel. Observation of the Borax data shows the pressure to be steadily increasing with time.

A reconciliation of this inconsistency could be based on the following argument. It is not known where the pressure transducer was located in the reactor. Assuming that it was a device of moderate size it would have been impossible to place it inside the reactor core. Or if it were placed inside the core the water channel at that location must have been much larger than those for the standard MTR fuel element. In any event the recorded pressure was probably not that existing at any time within a single standard water channel.

Depending on the type and orientation of the transducer its reading would be a combination of the static pressure existing at that point and the pressure caused by momentum

changes in the moving water close to the transducer pressure opening. If the momentum effects predominate, then the observed pressure is merely a measure of the velocity and amount of water being expelled by all of the water channels undergoing boiling at any time. Since the number of channels undergoing initial boiling is rapidly increasing with time, the amount of moving water is increasing with time and the observation of pressure increasing with time is consistent with this model.

It should be noted that the pressure trace of only one excursion was presented in the Borax report (Figure 43, Ref. 1). Mention was made of apparent inconsistencies in the pressure observations which were attributed to malfunctioning of the device. Careful scrutiny of this one pressure trace reveals something which could best be described as a jog in the curve at approximately the time of maximum fuel plate temperature. This could be attributed to the incidence of the overdeveloped boiling phase in the central channel at this time and the consequent new surge of expelled water.

H. Summary

The Borax experimental results and the investigations by Rosenthal, Whitehead and Rohsenow provided guides to the development of a physical model for the course of a runaway excursion in a water-moderated nuclear reactor. Within the framework of these guides it was the aim of this thesis to understand and analyze a reactor runaway excursion. The model developed is consistent with this aim.

The conduction phase, condition for initiation of boiling and condition of fully developed boiling were amenable to analytical investigation and the numerical results of these analyses agree with the available experimental evidence.

Analysis of the remaining phases of the developed physical model and a complete solution of the reactor excursion must await a more detailed and more fundamental understanding of transient boiling phenomena.

APPENDIX A

Table of Symbols

a	heat diffusivity of water $(\frac{ft^2}{Sec})$
A	fuel plate area (ft^2)
B	constant defined by equation (55)
C_ℓ	specific heat of water $(\frac{BTU}{lb\ ^\circ F})$
C	constant defined by equation (25)
h_{fg}	latent heat of vaporization of water $(\frac{BTU}{lb})$
K_ℓ	heat conductivity of water $(\frac{BTU}{Sec\ ft\ ^\circ F})$
δK	initial step increase in effective multiplication factor of the reactor
$\frac{\partial K}{\partial V}$	reactor void coefficient (ft^{-3})
ℓ	prompt neutron lifetime (Sec)
m	initial inverse reactor period defined by equation (2) (Sec^{-1})
P	power developed in a fuel plate per unit fuel plate area $(\frac{BTU}{Sec\ ft^2})$
$(\frac{Q}{A})$	rate of heat transfer per unit fuel plate area $(\frac{BTU}{Sec\ ft^2})$
S	heat capacity of fuel plates per unit fuel plate area $(\frac{BTU}{ft^2\ ^\circ F})$
t	time (Sec)
T	temperature $(^\circ F)$

T_b	fuel plate temperature at initiation of boiling ($^{\circ}F$)
V	reactor volume (ft^3)
V_v	vapor volume in a water channel per unit fuel plate area (ft)
x	distance into the water from and normal to a fuel plate (ft)
x'	distance into the water at which the slope of the temperature profile which existed at the instant of bubble formation equals the slope of the temperature profile at the outer edge of the boiling region at the time of fully developed boiling (ft)
α	boiling heat transfer coefficient defined by equation (18) $\left(\frac{BTU}{Sec\ ft^2\ {}^{\circ}F^3}\right)$
β	fraction of fission neutrons which are delayed
μ_l	viscosity of water $\left(\frac{lb}{Sec\ ft}\right)$
ω	statistical weight at a point for reactor perturbations
ϕ	flux at a point in the reactor $\left(\frac{neutrons}{cm^2\ Sec}\right)$
ρ	density $\left(\frac{lbs}{ft^3}\right)$
σ	surface tension of water $\left(\frac{lb}{ft}\right)$

Subscripts

b	boiling
i	initial
ℓ	liquid
m	maximum
o	point of maximum flux within the reactor
p	fuel plate
r	point r within the reactor
s	saturation
v	vapor

APPENDIX B

Transient one dimensional heat conduction into a semi infinite medium with an exponentially rising surface temperature

The problem can be expressed analytically in the following manner. The unsteady state diffusion differential equation is

$$(26) \quad \frac{\partial^2 T_e(x,t)}{\partial x^2} = \frac{1}{a} \frac{\partial T_e(x,t)}{\partial t}$$

and if the temperature scale is based upon the initial temperature, then initially

$$(27) \quad T_e(x,0) = 0$$

and the surface temperature is varying with time

$$(28) \quad T_e(0,t) = \phi(t)$$

then the solution can be obtained in the following way.

Let $T = F(x,t)$ be the solution for $t > 0$ in a case in which $T(o,t) = 0$ for $-\infty \leq t < 0$ and $T(o,t) = 1$ for $0 \leq t$. This solution is (Ref. 6)

$$(29) \quad F(x,t) = \frac{2}{\sqrt{\pi}} \int_{\frac{x}{2\sqrt{at}}}^{\infty} e^{-\beta^2} \, d\beta$$

Then if $T(o,t) = 0$ for $-\infty \leq t < \tau$ and $T(o,t) = 1$ for $\tau \leq t$ the solution for $t \geqslant \tau$ is

$$(30) \quad T(x,t) = F(x, t-\tau)$$

Similarly if $T(o,t) = 0$ for $-\infty \leq t < \tau + d\tau$ and $T(o,t) = 1$ for $\tau + d\tau \leq t$ the solution for $t \geqslant \tau + d\tau$ is

$$T(x,t) = F(x, t-\tau-d\tau) \tag{31}$$

Hence if $T(o,t) = 0$ for $-\infty \leq t < \tau$, and $T(o,t) = 1$ for $\tau \leq t < \tau + d\tau$, and $T(o,t) = 0$ for $\tau + d\tau \leq t$ the solution for $t \geqslant \tau$ is

$$T(x,t) = F(x,t-\tau) - F(x,t-\tau-d\tau) \tag{32}$$

or in the limit as $d\tau \to 0$

$$T(x,t) = \frac{\partial}{\partial t}\Big[F(x,t-\tau)\Big]\,d\tau \tag{33}$$

Now if instead of a surface temperature of 1 for this time interval $d\tau$ at $t = \tau$ we have a surface temperature of $\phi(\tau)$, then the solution to this problem would be

$$T(x,t) = \phi(\tau)\,\frac{\partial}{\partial t}\Big[F(x,t-\tau)\Big]\,d\tau \tag{34}$$

We may now sum up the results for each time interval d over the total interval $t = 0$ to $t = t$ and find the solution for the surface temperature of $\phi(t)$

$$T(x,t) = \int_0^t \phi(\tau)\,\frac{\partial}{\partial t}\Big[F(x,t-\tau)\Big]\,d\tau \tag{35}$$

From equation (29)

$$F(x, t-\tau) = \frac{2}{\sqrt{\pi}} \int_{\frac{x}{2\sqrt{a(t-\tau)}}}^{\infty} e^{-\beta^2}\,d\beta \tag{36}$$

Therefore

$$(37)\quad \frac{\partial}{\partial t}\Big[F(x,t-\tau)\Big] = -\frac{2}{\sqrt{\pi}}\, e^{-\frac{x^2}{4a(t-\tau)}}\, \frac{\partial}{\partial t}\left[\frac{x}{2\sqrt{a(t-\tau)}}\right]$$

$$= \frac{x}{2\sqrt{\pi a\,(t-\tau)^3}}\; e^{-\frac{x^2}{4a(t-\tau)}}$$

Substituting in equation (35)

$$(38)\quad T(x,t) = \frac{x}{2\sqrt{\pi a}} \int_0^t \phi(\tau)\, \frac{e^{-\frac{x^2}{4a(t-\tau)}}}{(t-\tau)^{3/2}}\, d\tau$$

To simplify let

$$(39)\quad \mu = \frac{x}{2\sqrt{a(t-\tau)}}$$

Substituting in equation (38)

$$(40)\quad T(x,t) = \frac{2}{\sqrt{\pi}} \int_{\frac{x}{2\sqrt{at}}}^{\infty} \phi\left(t - \frac{x^2}{4\mu^2 a}\right) e^{-\mu^2}\, d\mu$$

Now if from equation (5)

$$(5)\quad T_P - T_i = \frac{P_i}{Sm}\, e^{mt}$$

Then substituting in equation (40)

$$(41)\quad T_e(x,t) - T_i = \frac{2}{\sqrt{\pi}}\, \frac{P_i}{Sm}\, e^{mt} \int_{\frac{x}{2\sqrt{at}}}^{\infty} e^{-\left(\frac{x^2}{4a\mu^2} + \mu^2\right)}\, d\mu$$

The solution of which can be easily found using Laplace transforms (Ref. 6) to be

$$(42) \quad T_\ell(x,t) - T_i = \frac{P_i}{2Sm} e^{mt} \left\{ e^{-x\sqrt{\frac{m}{a}}} \operatorname{erfc}\left[\frac{x}{2\sqrt{at}} - \sqrt{mt}\right] + e^{x\sqrt{\frac{m}{a}}} \operatorname{erfc}\left[\frac{x}{2\sqrt{at}} + \sqrt{mt}\right] \right\}$$

For times greater than several exponential periods the transient terms in this solution die out and the solution approaches an asymptotic solution given by

$$(43) \quad T_\ell(x,t) - T_i = \frac{P_i}{Sm} e^{mt} e^{-x\sqrt{\frac{m}{a}}}$$

which is equation (6).

APPENDIX C

Development of analytical criterion for initiation of boiling

The temperature gradient into the water at the time the plate temperature is T_p is given by equation (7).

$$T_\ell(x, T_P) - T_i = (T_P - T_i)\, e^{-x\sqrt{\frac{m}{a}}} \tag{7}$$

Expanding the exponential term and keeping only the first two terms we get approximately

$$T_\ell(x) - T_i \cong (T_P - T_i)\left(1 - x\sqrt{\frac{m}{a}}\right) \tag{44}$$

Using saturation temperature for a base we get

$$T_\ell(x) - T_S = (T_P - T_S) - (T_P - T_i)\, x\sqrt{\frac{m}{a}} \tag{45}$$

or in spherical coordinates with the origin a point on the plate surface

$$T_\ell(x) - T_S = (T_P - T_S) - (T_P - T_i)\sqrt{\frac{m}{a}}\, r\cos\theta \tag{46}$$

A differential volume in spherical coordinates is given by

$$dV = 2\pi r^2 \sin\theta\, dr\, d\theta \tag{47}$$

Therefore the total enthalpy with saturation temperature as a base contained in a hemispherical volume of water of radius R centered at a point on the plate surface is given by

$$(48)\quad \Delta H = 2\pi C_\ell \rho_\ell \int_0^R \int_0^{\pi/2} \left[(T_P - T_S) - (T_P - T_i)\sqrt{\frac{m}{a}}\, r\cos\theta\right] r^2 \sin\theta \, dr\, d\theta$$

or

$$(49)\quad \Delta H = \frac{2}{3}\pi C_\ell \rho_\ell \left[(T_P - T_S)R^3 - \frac{3}{8}(T_P - T_i)\sqrt{\frac{m}{a}}\, R^4\right]$$

and the maximum Δ H occurs at the point where

$$(50)\quad \frac{\partial \Delta H}{\partial R} = 0 = \frac{2}{3}\pi C_\ell \rho_\ell \left[3R_m^2 (T_P - T_S) - \frac{3}{2}R_m^3 (T_P - T_i)\sqrt{\frac{m}{a}}\right]$$

or

$$(51)\quad R_m = 2\left(\frac{T_P - T_S}{T_P - T_i}\right)\sqrt{\frac{a}{m}}$$

Substituting equation (51) into equation (49) and simplifying

$$(52)\quad \Delta H_m = \frac{4}{3}\pi C_\ell \rho_\ell \left(\frac{a}{m}\right)^{\frac{3}{2}} \frac{(T_P - T_S)^4}{(T_P - T_i)^3}$$

Now this maximum energy is assumed equal to the energy required to supply the latent heat of vaporization necessary to form a hemispherical bubble of a radius r.

Or

$$(53)\quad \Delta H_m = \frac{2}{3}\pi r^3 h_{fg} \rho_v$$

Equating equations (52) and (53) and simplifying we get for the criterion of plate temperature at the initiation of boiling

$$(54) \qquad B\sqrt{m} = \frac{(T_b - T_s)^{4/3}}{(T_b - T_i)}$$

which is equation (8), and where

$$(55) \qquad B = \frac{r}{\sqrt{a}} \left(\frac{h_{fg}\, \rho_v}{2\, c_\ell\, \rho_\ell} \right)^{1/3}$$

Using values for water at atmospheric pressure and saturation temperature and an assumed bubble radius of 3 mils

$r = 3 \text{ mils} = 2.50 \times 10^{-4}$ ft

$a = 1.322 \times 10^{-3}$ ft sec$^{-\frac{1}{2}}$

$h_{fg} = 970$ BTU/lb

$\rho_v = 3.73 \times 10^{-2}$ lbs/ft^3

$\rho_\ell = 59.7$ lbs/ft^3

$c_\ell = 1.0$ BTU/lb$^\circ$F

we get

$B = 0.1271 \quad {}^\circ\text{F}^{1/3} \quad \text{sec}^{\frac{1}{2}}$

This compares with the experimentally determined value from Rosenthal's one experimental point (Ref. 3) of

$B = 0.1147 \quad {}^\circ\text{F}^{1/3} \quad \text{sec}^{\frac{1}{2}}$.

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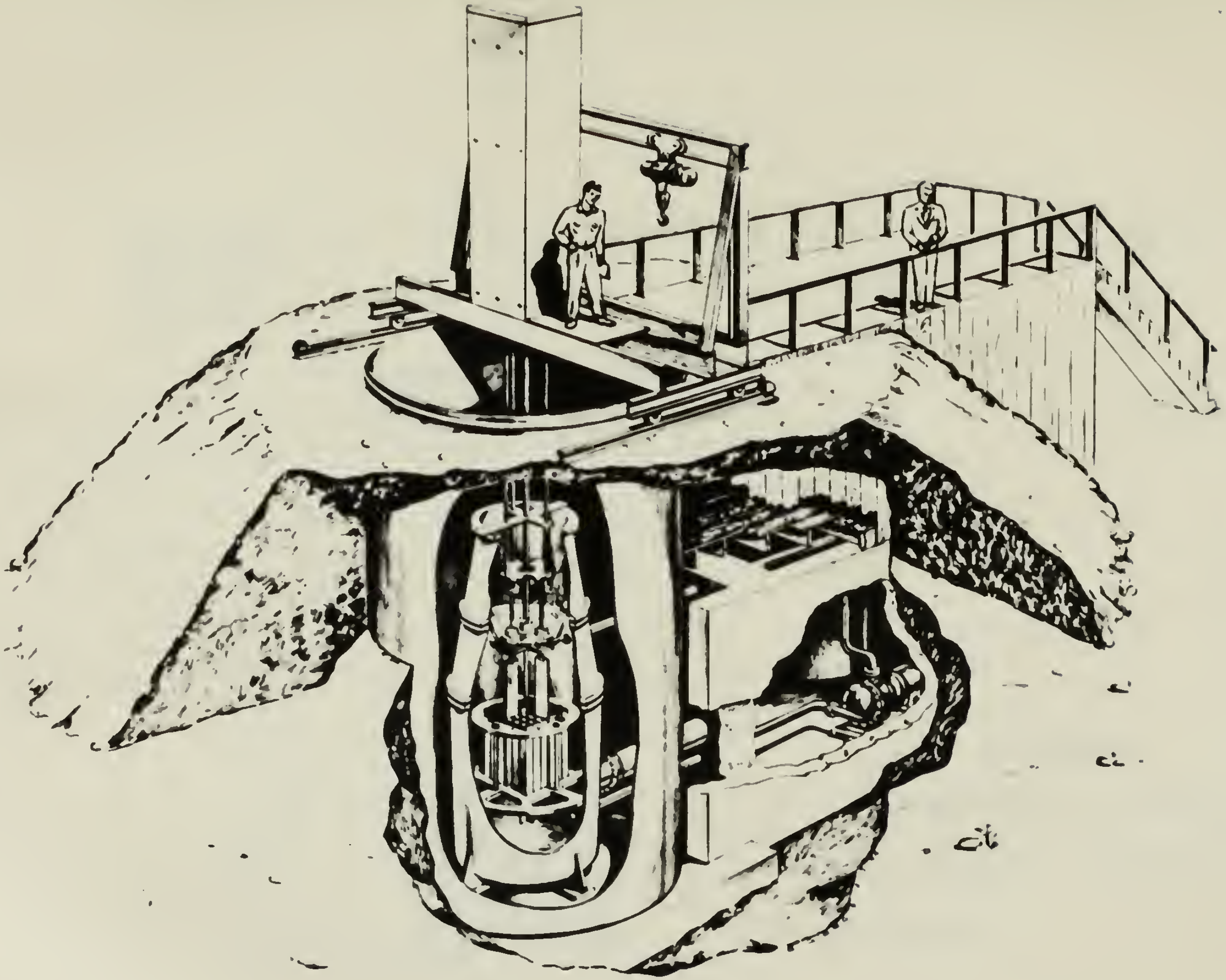


FIG. I

CUTAWAY DRAWING OF BORAX INSTALLATION

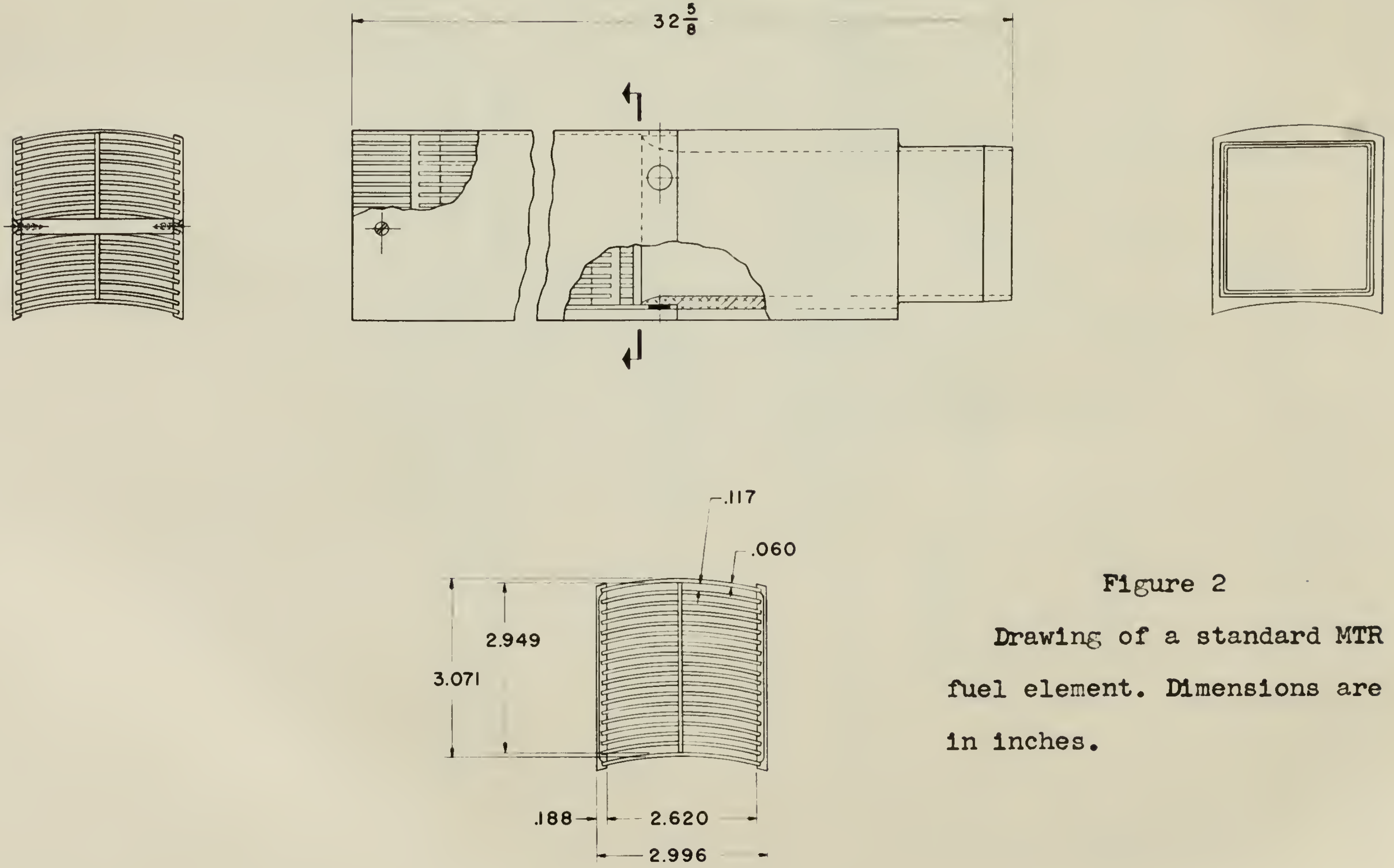
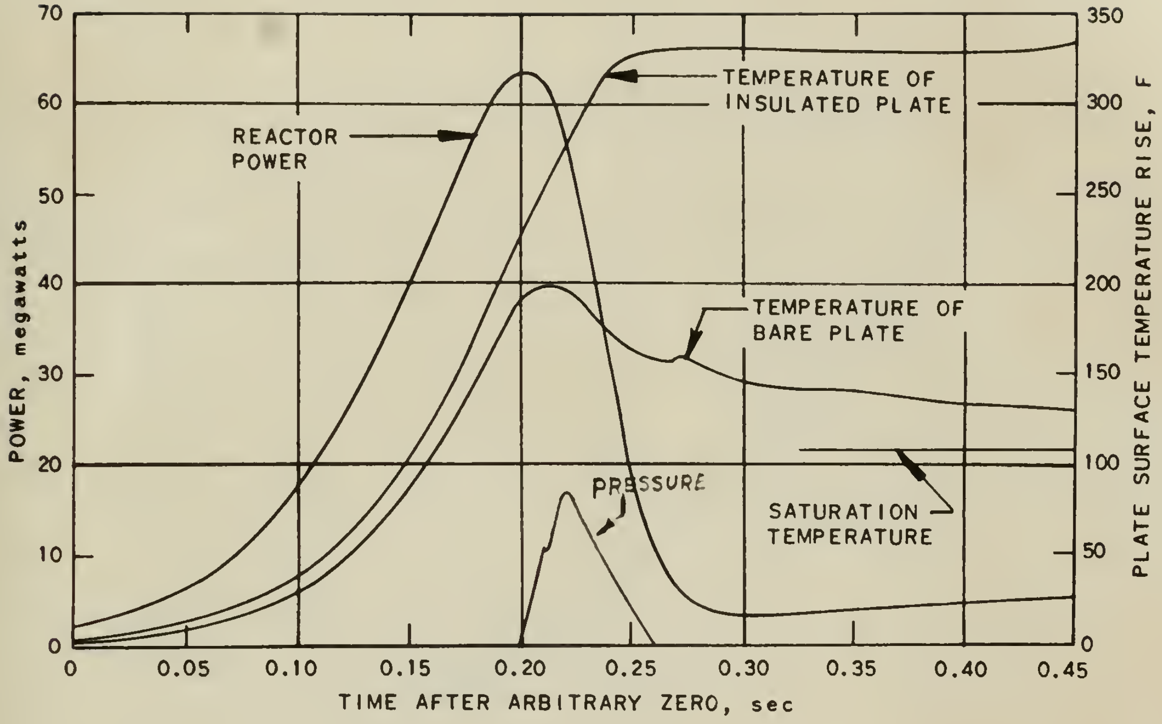


Figure 2

Drawing of a standard MTR fuel element. Dimensions are in inches.



PERIOD = 0.051 sec
INITIAL TEMPERATURE = 82 F

Figure 3

Plot of power, fuel plate temperature and pressure rise during a typical nuclear runaway excursion of the Borax reactor.

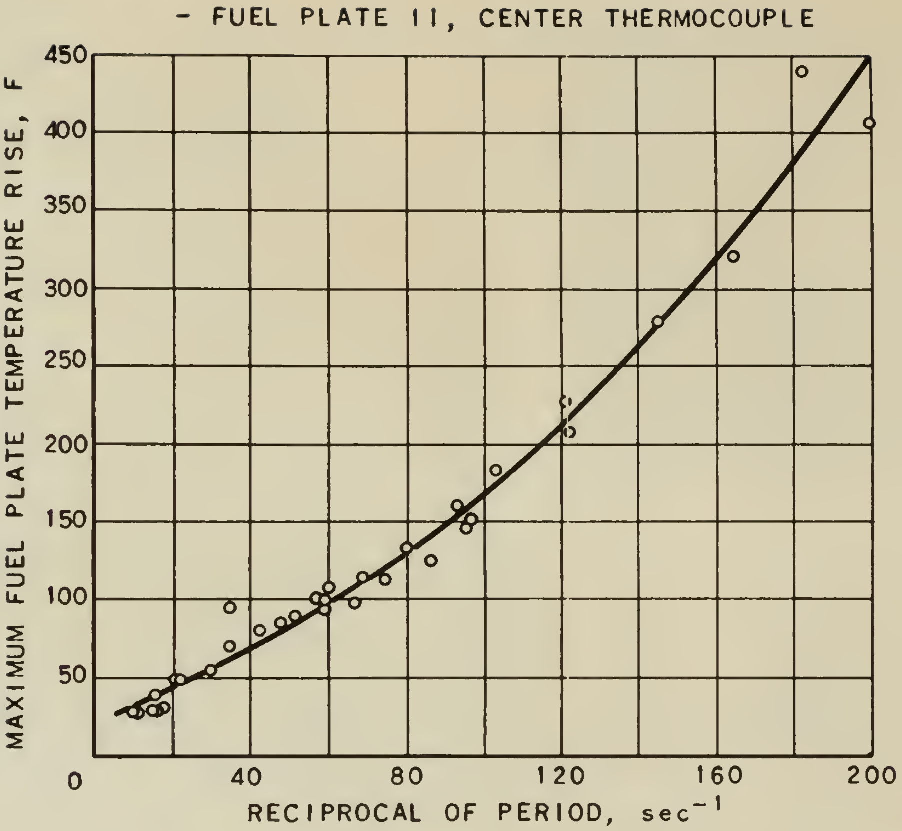


Figure 4

Borax maximum plate temperature rise for runaway excursions of various periods, with reactor initially at saturation temperature.

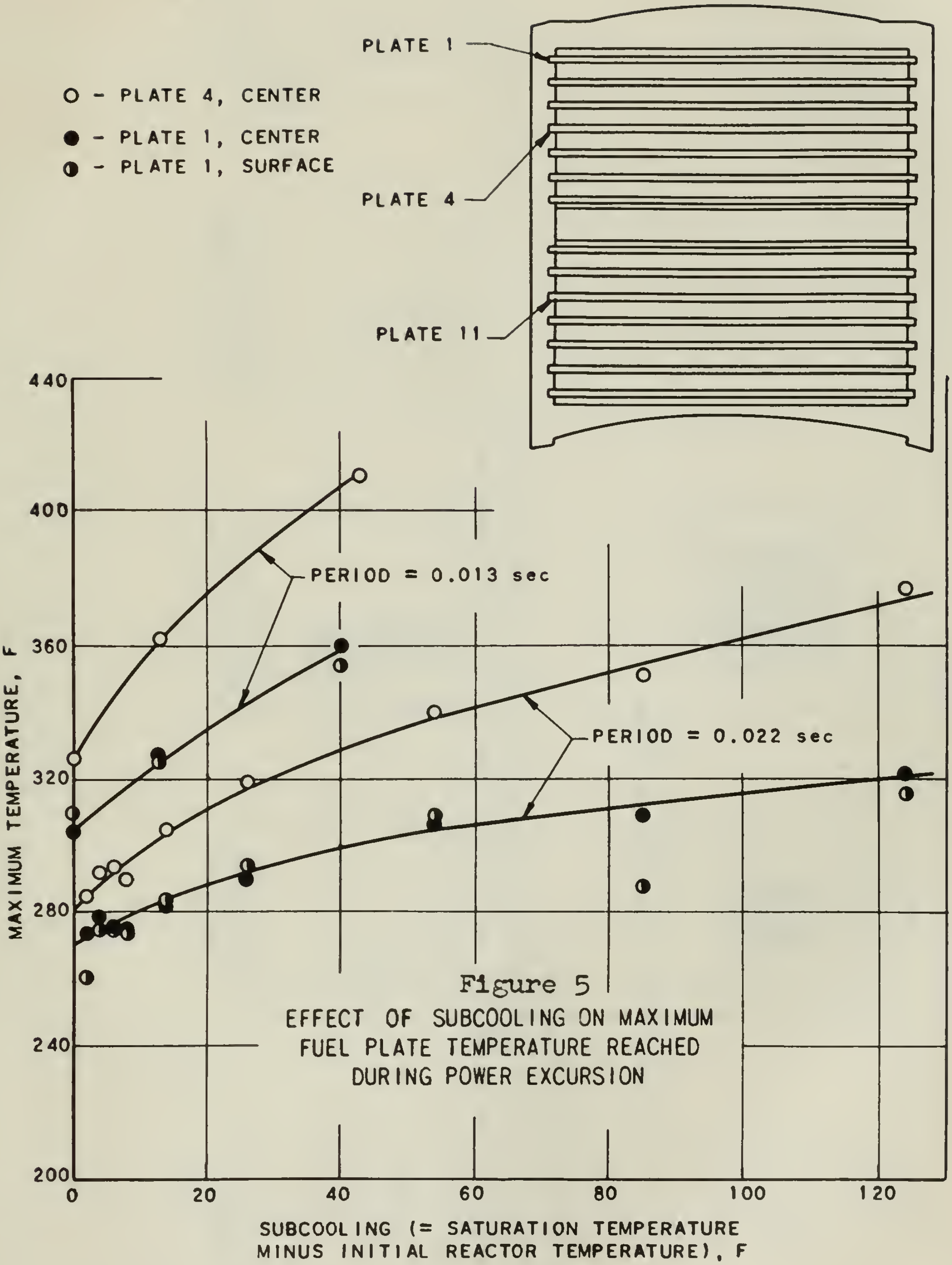
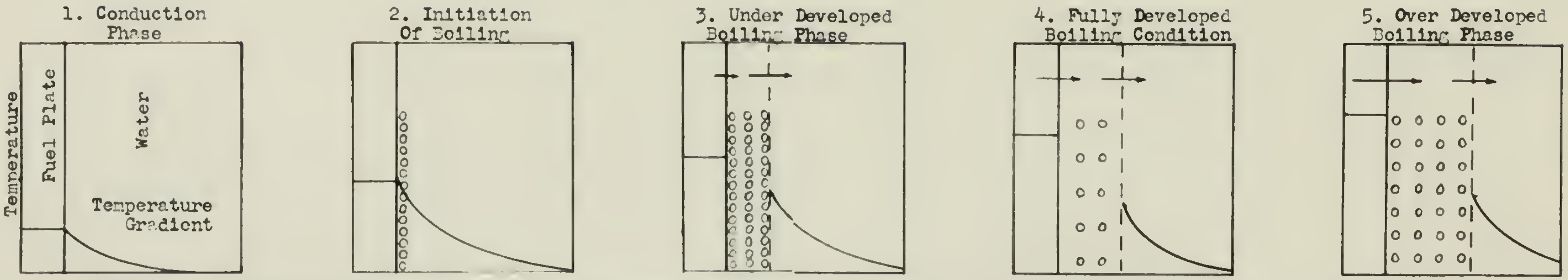
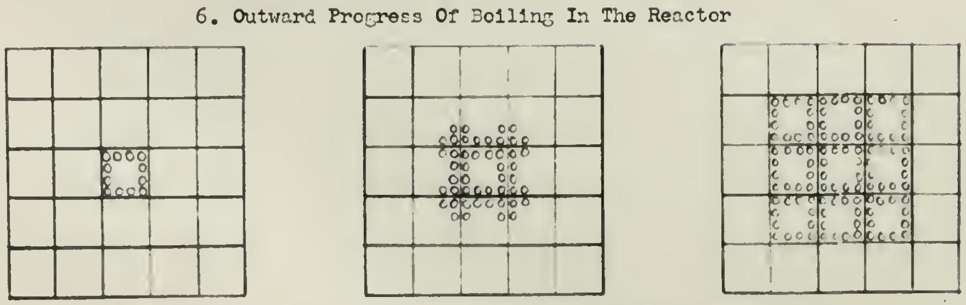


Figure 5

EFFECT OF SUBCOOLING ON MAXIMUM FUEL PLATE TEMPERATURE REACHED DURING POWER EXCURSION


1. Conduction Phase
Temperature
Fuel Plate
Water
Temperature Gradient
2. Initiation Of Boiling
3. Under Developed Boiling Phase
4. Fully Developed Boiling Condition
5. Over Developed Boiling Phase

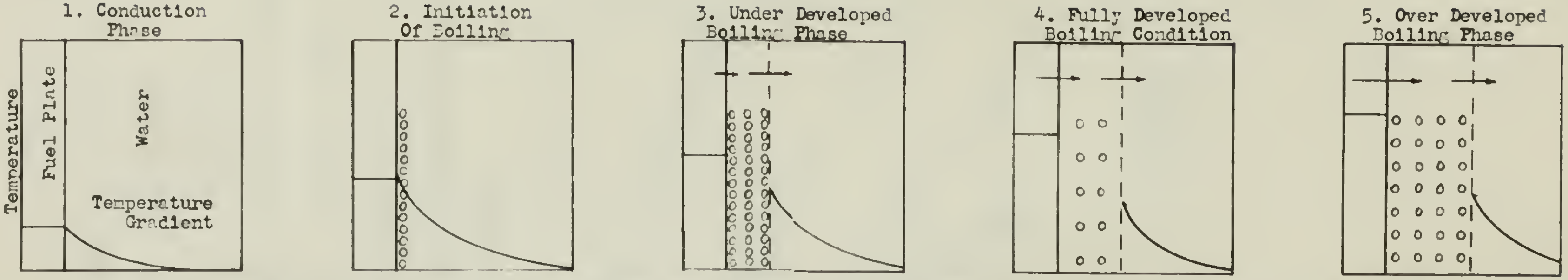
Schematic representation of a fuel plate and adjacent water channel: Temperature scale is vertical; Solid curve is temperature profile; The number of circles present indicate the intensity of boiling; And the length of the arrow at any point indicates the rate of heat transfer at that point.



Schematic representation of a horizontal section through the center of the reactor showing fuel plates and water channels.

Figure 6

Pictorial representation of successive physical events occurring within a reactor in a runaway excursion.



1. Conduction Phase

2. Initiation Of Boiling

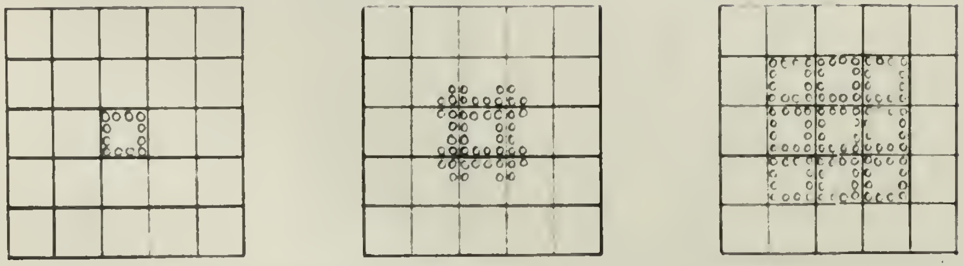
3. Under Developed Boiling Phase

4. Fully Developed Boiling Condition

5. Over Developed Boiling Phase

Schematic representation of a fuel plate and adjacent water channel: Temperature scale is vertical; Solid curve is temperature profile; The number of circles present indicate the intensity of boiling; And the length of the arrow at any point indicates the rate of heat transfer at that point.

6. Outward Progress Of Boiling In The Reactor



Schematic representation of a horizontal section through the center of the reactor showing fuel plates and water channels.

Figure 6

Pictorial representation of successive physical events occurring within a reactor in a runaway excursion.

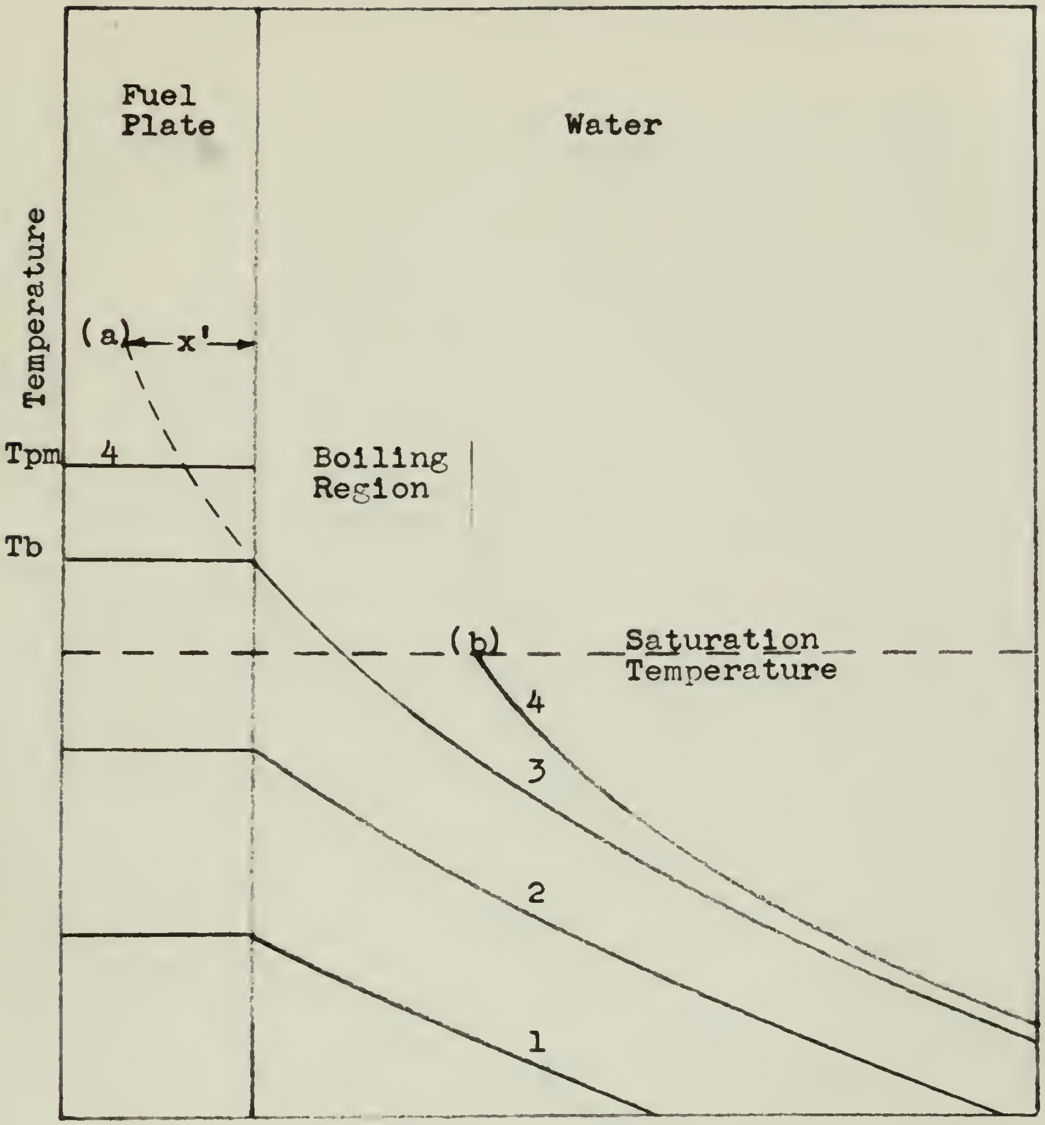


Figure 7

Schematic representation of temperature profiles occurring in a fuel plate and adjacent water channel at various times during a reactor runaway excursion.

1 and 2 Conduction phase

3 Initiation of boiling

4 Fully developed boiling

Note that the slope of profile 3 at (a) is the same as profile 4 at (b).

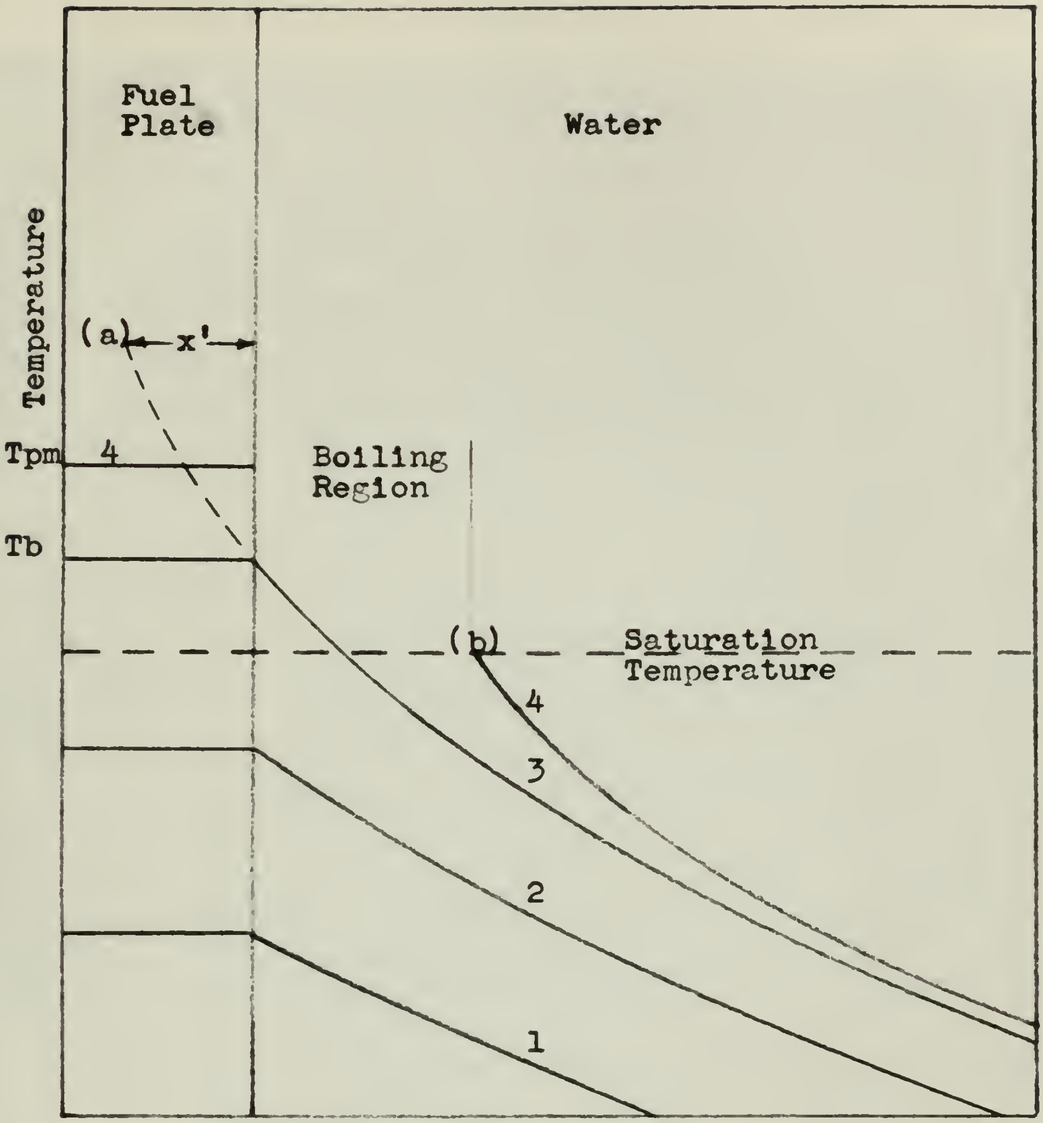


Figure 7

Schematic representation of temperature profiles occurring in a fuel plate and adjacent water channel at various times during a reactor runaway excursion.

1 and 2	Conduction phase
3	Initiation of boiling
4	Fully developed boiling

Note that the slope of profile 3 at (a) is the same as profile 4 at (b).

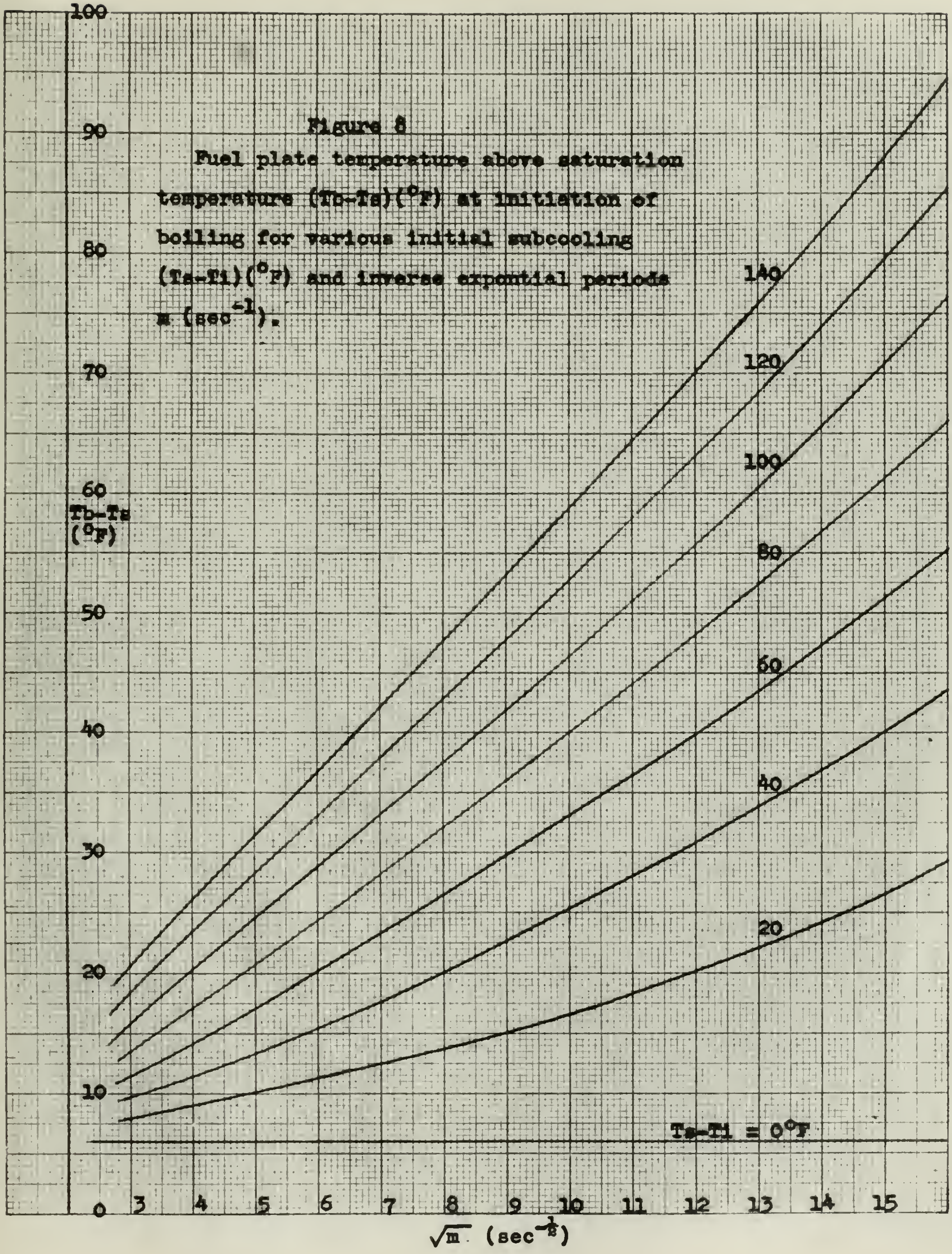


Figure 8

Fuel plate temperature above saturation temperature (Tb-Ts)(°F) at initiation of boiling for various initial subcooling (Ts-Ti)(°F) and inverse expontial periods m (sec^{-1}).

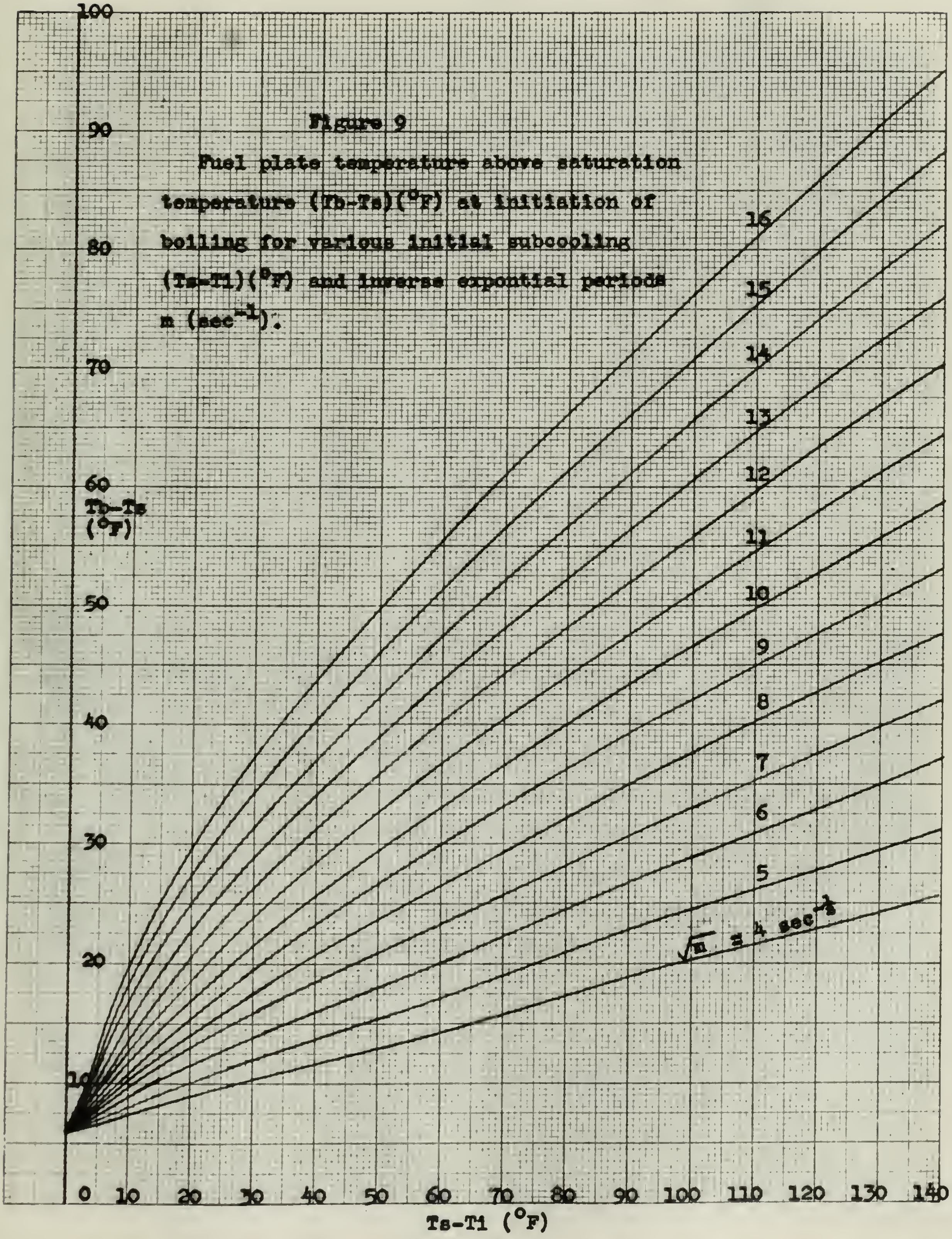


Figure 9

Fuel plate temperature above saturation temperature (Tb-Ts)(°F) at initiation of boiling for various initial subcooling (Ts-Ti)(°F) and inverse expontial periods m (sec^{-1}).

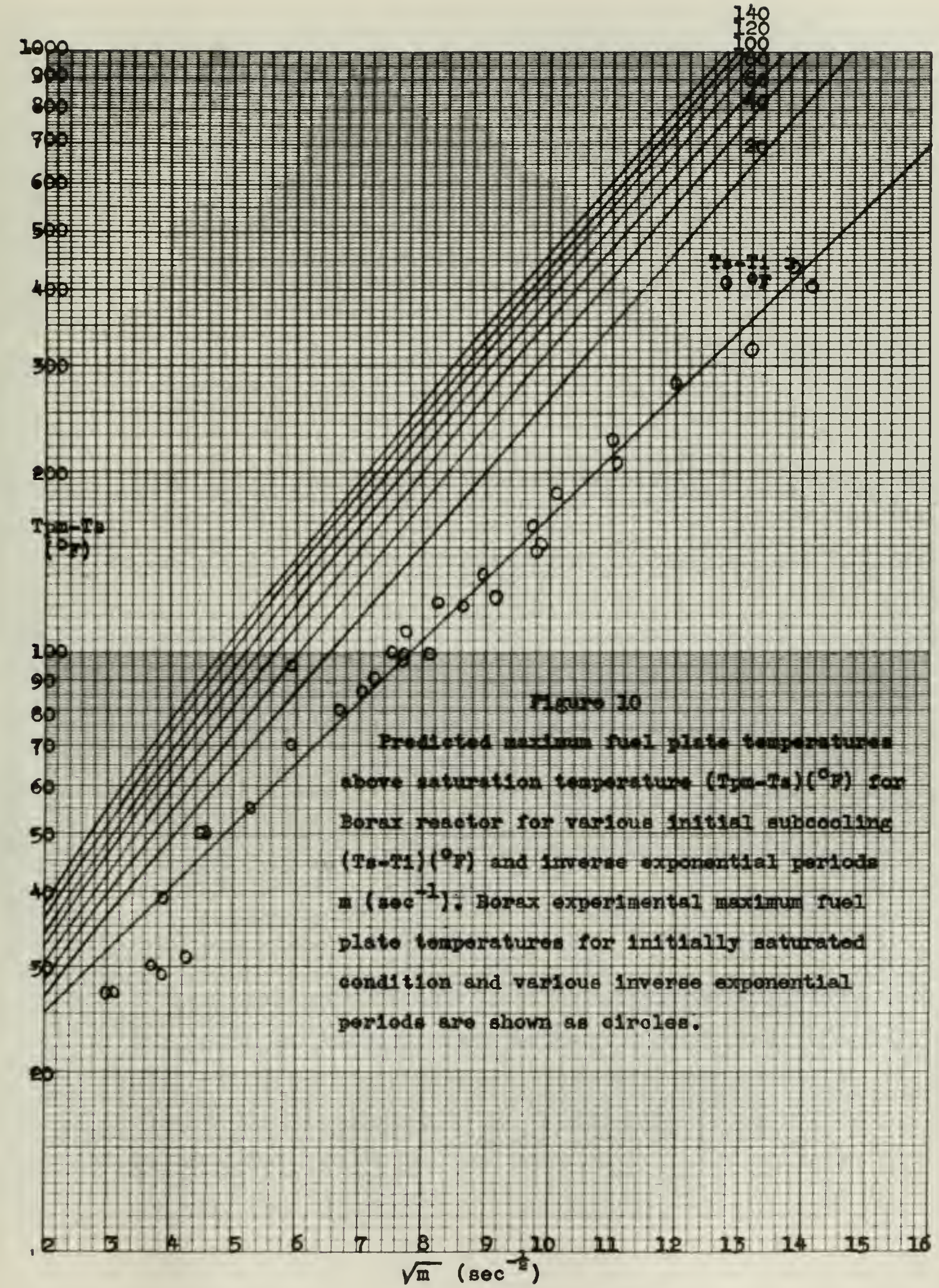


Figure 10

Predicted maximum fuel plate temperatures above saturation temperature (Tpm-Ts)(°F) for Borax reactor for various initial subcooling (Ts-Ti)(°F) and inverse exponential periods m (sec^{-1}). Borax experimental maximum fuel plate temperatures for initially saturated condition and various inverse exponential periods are shown as circles.

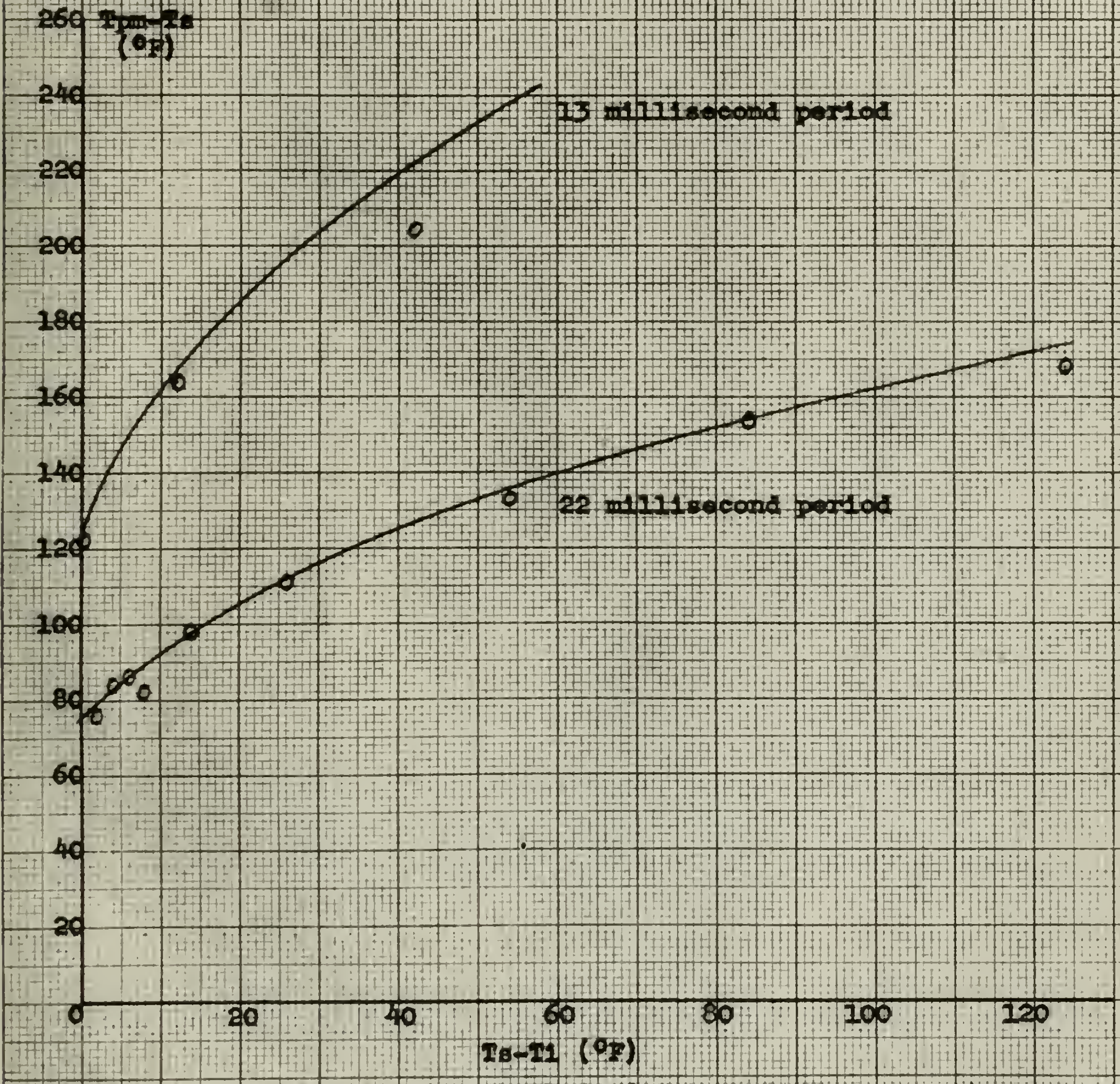


Figure 11

Comparison of predicted with Borax experimental maximum fuel plate temperatures above saturation temperature (Tpm-Ts) (°F) for 13 and 22 millisecond exponential periods and various initial subcooling (Ts-Ti)(°F). Circles are experimental data.

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